Available online at www.sciencedirect.com
ScienceDirect
International Journal of Heat and Fluid Flow 29 (2008) 75–82
International Journal of HEAT AND FLUID FLOW
www.elsevier.com/locate/ijhff

The influence of swirl on the reattachment length in an abrupt axisymmetric expansion

M. Vanierschot *, E. Van den Bulck [1]

Katholieke Universiteit Leuven, Department of Mechanical Engineering, Celestijnenlaan 300A, B-3001 Heverlee, Belgium

Received 21 November 2006; received in revised form 17 September 2007; accepted 18 September 2007
Available online 30 October 2007

Abstract

This paper presents a theoretical study on the influence of swirl on the reattachment length in an abrupt axisymmetric expansion. A scale analysis of the equations of motion reveals that this reattachment length decreases with increasing swirl. Moreover, it seems to suggest that there may be a singularity in the non-swirling sudden expansion flow, which disappears with increasing swirl beyond a critical value. This singularity is a possible explanation for the scatter of the data found in the literature at zero swirl. For the swirling flow, an analytical expression for the reattachment length is derived based on similarities found in experimental data between swirling and non-swirling flows. This expression shows that the reattachment length is a function of the swirl number and the expansion ratio and, to some extend, of the reattachment length at zero swirl. It is found that the local swirl number of the detaching streamline S_D is a better parameter to characterise the reattachment length than the momentum flux averaged swirl number S. The theoretical model is validated based on values for the reattachment length found in the literature for expansion ratios $h = R_2/R_1$ ranging from 1.5 to 2, Reynolds numbers Re from 10,000 to 100,000 and swirl numbers S from 0 to 1.23. Comparison with experiments shows a good agreement between the model and experimental data.

Keywords: Swirling flow; Sudden expansion; Reattachment length

1. Introduction

Separation of flows in channels with a sudden expansion are well reported in the literature. The influence of both the expansion ratio and Reynolds number on the reattachment length are well documented, both for 2D backward facing steps (Armaly et al., 1983) and axisymmetric sudden expansions (Back and Roschke, 1972; Zemanick and Dougall, 1970; Morrison et al., 1988; Amano, 1982). Back and Roschke (1972) performed dye studies to observe the influence of inlet Reynolds number on the reattachment length in an abrupt axisymmetric expansion with an expansion ratio of 2.6. They found that for high Reynolds numbers the reattachment length remains more or less constant at 9 step heights. It is confirmed by other investigators (Zemanick and Dougall, 1970) that, in the high Reynolds number regime, the reattachment length expressed in step heights is independent of both the Reynolds number and the expansion ratio. Values that are found in the literature for the reattachment point in the high Reynolds number regime for sudden pipe expansions can vary from 5 to 9 step heights downstream of the expansion. Eaton and Johnston (1980) suggest that part of the variation in cases with similar inlet conditions is due to the oscillation of the reattachment point because of the dynamic shrinking and growing of the recirculation zone. Kuehn (1980) suggests that the variation in reattachment length was attributed to adverse pressure gradient effects and differences in inlet conditions.

The study of the influence of swirl on the reattachment length is more limited. In the literature it is well known that increasing the swirl decreases the reattachment length.

* Corresponding author. Tel.: +32 16 322548; fax: +32 16 322985.
E-mail addresses: Maarten.Vanierschot@mech.kuleuven.be (M. Vanierschot), Eric.VandenBulck@mech.kuleuven.be (E. Van den Bulck).
[1] Prof. dr. ir., Head of the applied flow and combustion section.

0142-727X/$ - see front matter
doi:10.1016/j.ijheatfluidflow.2007.09.004

Nomenclature

A, B, C, D	proportionality factors in scale analysis
h	expansion ratio, R_2/R_1
x, y, z	cartesian coordinates
$\mathbf{C}$	circular channel
r, θ, z	cylindrical coordinates
$\mathscr{P}_i(j)$	projection of j on i
R	radius
Re	Reynolds number
S	swirl number
S_{D}	local swirl number
$S_{\psi_1^\alpha}$	local swirl number at the detaching streamline
t	dimensionless coordinate along the expansion line
u	local mean axial velocity
u_{D}	local mean axial velocity at detachment point
$u_{\psi_1^\alpha}$	local mean axial velocity at the detaching streamline
u_{zr}^α	projection of local mean axial velocity at the detaching streamline on a (z,r)-plane
U	mean axial velocity in the upstream channel
v	local mean radial velocity
w	local mean tangential velocity
W	mean tangential velocity in the upstream channel
z_{R}	reattachment length

Greek symbols

α	local swirl angle
δ	reattachment length in number of step heights, $z_{\mathrm{R}}/(R_2 - R_1)$
ψ	streamline
ψ_1	detaching streamline non-swirling flow
ψ^α	streamline swirling flow
ψ_1^α	detaching streamline swirling flow
$\psi_{\mathbf{C}_2}^\alpha$	projection of detaching streamline on the downstream channel
ψ_{zr}^α	projection of detaching streamline on a (z,r)-plane
$\psi_{1,L}$	expansion line non-swirling flow
$\psi_{1,L}^\alpha$	expansion line swirling flow
Ψ	stream-surface non-swirling flow
Ψ_1	detaching stream-surface non-swirling flow
Ψ^α	stream-surface swirling flow
Ψ_1^α	detaching stream-surface swirling flow
ρ	density

Subscripts

0	without swirl
D	refers to the detachment point
R	refers to the reattachment point
1	refers to the upstream channel
2	refers to the downstream channel
ψ	values along ψ

However, most studies of sudden expansion swirling flows focus on the entire flow field and the reattachment length is either not measured or reported upon only qualitatively (Hallett and Gunther, 1984; Abujelala et al., 1984; Garkusha and Kucherenko, 1980). Henceforth few quantitative data are available on reattachment lengths with varying swirl. An extensive experimental study is reported by Dellenback et al. (1988). They describe axial and tangential velocity profiles in an abrupt expansion with expansion ratio 1.94. In their study the Reynolds number is varied from 30,000 to 100,000 and the swirl number from 0 to 1.23. Quantitative data for the reattachment length with different swirl number are given and it is found that the reattachment length decreases with increasing swirl. These results agree with other researchers (Rhode et al., 1983; Ahmed and Nejad, 1992; Ahmed, 1998). Another detailed study is published by Wang et al. (2004). They report LDA measurements in combination with LES simulations of a sudden expansion of ratio 1.94 in a model dumb combustor. The investigated swirl numbers are $S = 0$, 0.33 and 0.43. Detailed mean velocity fields in three directions are reported from which the reattachment length can be derived directly.

In this paper an analytical expression for the influence of swirl on the reattachment length in a sudden axisymmetric expansion is derived. A scale analysis of the equations of motion shows that this reattachment length decreases with increasing swirl. First some considerations and definitions in non-swirling flows in sudden expansions are derived which serve as a basis for the model for swirling flows. The validation is based on data of the reattachment length of swirling flows found in the literature (Dellenback et al., 1988; Favaloro et al., 1989; Wang et al., 2004; Ahmed and Nejad, 1992; Nejad and Ahmed, 1992). It is found that for Reynolds numbers ranging from 10,000 to 100,000 and expansion ratios from 1.5 to 2, the model gives good predictions of the reattachment length.

2. Theoretical considerations and definitions

2.1. No swirl

A schematic view of the sudden expansion flow is shown in Fig. 1. Consider a fully developed non-swirling flow with a mean velocity U through a circular channel with radius

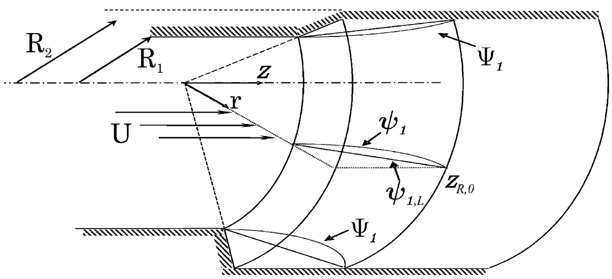

Fig. 1. Schematic view of the axisymmetric sudden expansion without swirl. Ψ denotes an axisymmetric 'stream-surface' while ψ denotes a streamline.

R_1 (upstream channel). A sudden expansion to a circular channel with radius R_2 (downstream channel) is situated at $z = 0$. The expansion ratio is defined as $h = R_2/R_1$. Both the channels are axisymmetric and their axes of symmetry are coincident with the z-axis in a polar (r, θ, z) coordinate system. Due to the axisymmetry, the flow is 2-dimensional in the (z, r)-plane. The flow separates and creates a recirculation zone behind the sudden expansion. The flow detaches at the sudden expansion with coordinates $(R_1, \theta, 0)$ with $-\pi \leqslant \theta \leqslant \pi$, called the detachment circle. The flow reattaches to the downstream channel at coordinates $(R_2, \theta, z_{R,0})$ with $-\pi \leqslant \theta \leqslant \pi$, called the reattachment circle. Consider now the normalised streamfunction $\Psi(r, z)$ in polar coordinates defined as

$$\Psi(r,z) = \int_0^r u(r,z) r \, dr \bigg/ \int_0^{R_1} u(r,z) r \, dr. \quad (1)$$

The projection of the velocity vector in a meridional (z, r)-plane is directed along the axisymmetric 'stream-surfaces' $\Psi(r, z) = \text{const.}$, and the fluid particle paths are lines along these surfaces. The stream-surface at the detachment circle can be written in an implicit form as $|\Psi(r, z)| = 1$, further on called Ψ_1. The streamlines at the sudden expansion, called ψ_1, are 2D curves along Ψ_1 as in Fig. 1.

2.2. Swirling flow

In the case of swirling flows, the dynamics of the flow can be very different and the mean (turbulence averaged) swirling flow in a sudden expansion may not necessarily be time independent. At certain swirl numbers, a large coherent structure is present with an oscillating character Gupta et al. (1984). This structure is referred to as the 'Precessing Vortex Core' or PVC. An extensive overview of the studies regarding the PVC is given by Syred (2006). When these periodic fluctuations are time averaged, the mean flow field becomes axisymmetric and the streamfunction is still given by Eq. (1). Fig. 2 shows a schematic view of the sudden expansion flow with swirl. The stream-surface at the detachment point in the swirling flow is given by $|\Psi^\alpha(r, z)| = 1$, further on called Ψ_1^α. The flow detaches at the sudden expansion with coordinates $(R_1, \theta, 0)$ with $-\pi \leqslant \theta \leqslant \pi$ and the flow reattaches to the downstream

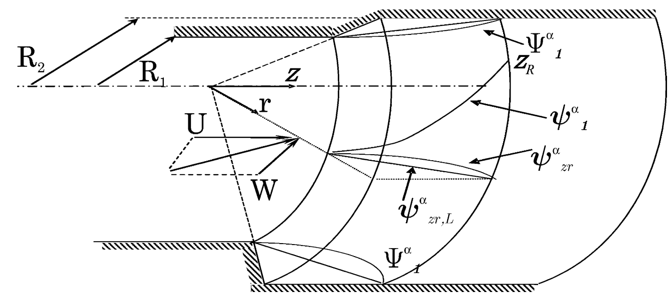

Fig. 2. Schematic view of the axisymmetric sudden expansion with swirl. Ψ denotes an axisymmetric 'stream-surface' while ψ denotes a streamline.

channel at coordinates (R_2, θ, z_R) with $-\pi \leqslant \theta \leqslant \pi$, where z_R is the reattachment length. In contrast with a non-swirling flow, the streamline ψ_1^α is no longer a 2D curve due to the tangential velocity component and is therefore a function of the rotational angle θ. The streamline ψ_1^α is a spiral along Ψ_1^α. Let us now define 2 projections of ψ_1^α. The first one is the projection along the θ-direction in a (z, r)-plane called $\psi_{zr}^\alpha = \mathscr{P}_{zr}(\psi_1^\alpha)$ and the second one is the projection along the radial direction on the outer cylinder called $\psi_{\mathbf{C}_2}^\alpha = \mathscr{P}_{\mathbf{C}_2}(\psi_1^\alpha)$, where $\mathbf{C}_2$ is a surface representing channel 2.

The local swirl number $S_{\Psi_1^\alpha}$ along a streamline of detachment ψ_1^α is defined as

$$S_{\Psi_1^\alpha} = \frac{w_{\psi_1^\alpha}}{u_{\psi_1^\alpha}}, \quad (2)$$

where $w_{\psi_1^\alpha}$ and $u_{\psi_1^\alpha}$ are the time mean tangential and axial velocities along ψ_1^α. The local swirl angle α is defined as $\arctan(w_{\psi_1^\alpha}/u_{\psi_1^\alpha})$. The curves of $S_{\Psi_1^\alpha}$ as a function of the dimensionless z-coordinate along Ψ_1^α are shown in Fig. 3. These values are obtained from the quantitative experimental data of Dellenback (1986) and Wang et al. (2004). The reported velocity measurements have an accuracy of 3% relative error and their position has a relative error of 1% in the experiment of Dellenback. The relative error of Ψ^α,

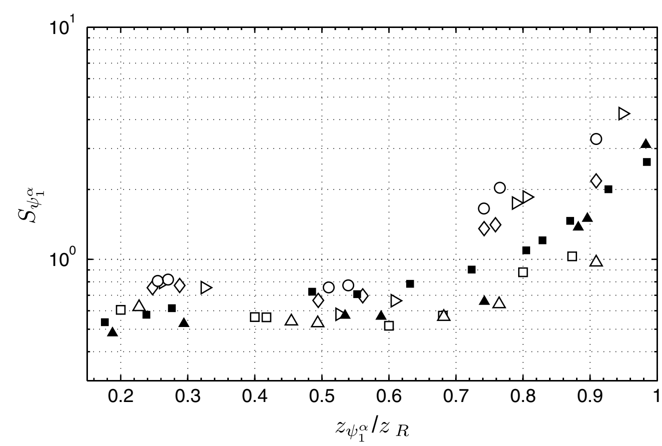

Fig. 3. The local swirl number $S_{\psi_1^\alpha}$ as a function of the dimensionless z-coordinate of ψ_1^α. Data from Dellenback (1986). □: $S = 0.6$, $Re = 30{,}000$; △: $S = 0.74$, $Re = 100{,}000$; ▷: $S = 0.98$, $Re = 30{,}000$; ◇: $S = 1.16$, $Re = 60{,}000$; ○: $S = 1.23$, $Re = 100{,}000$. Data from Wang et al. (2004). ▲: $S = 0.33$, $Re = 10{,}000$; ■: $S = 0.43$, $Re = 20{,}000$.

given by Eq. (1), is estimated to be 4% and the relative error of the position of Ψ_1^α is 4%. Based on these errors the relative error of the local swirl number $S_{\Psi_1^\alpha}$ is 7%. From the number of samples collected by Wang et al. the relative error of the velocity measurements is estimated to be maximum 1% in a 99% confidence interval. The relative error of Ψ and its position is estimated to be 3%. Based on these errors the relative error of the local swirl number is 6%. The error bars in Fig. 3 are not shown in order to not overload the figure. Data are only available for $z_{\psi_1^\alpha}/z_R \geqslant 0.2$. The experimental data show that $S_{\psi_1^\alpha}$ remains constant for $z_{\psi_1^\alpha}/z_R \leqslant 0.6$ and starts to increase rapidly further along the streamline because $\lim_{r\to R_2} u_{\psi_1^\alpha}(r)$ goes faster to zero than $\lim_{r\to R_2} w_{\psi_1^\alpha}(r)$. In the reattachment point the ratio is undefined. The local swirl number at the detachment point S_D is estimated as the constant value in the beginning of the curves in Fig. 3.

2.3. Scale analysis

The steady state equations of continuity and motion in the z- and r-direction for an axisymmetric inviscid fluid are written as

$$\frac{\partial u}{\partial z}+\frac{\partial v}{\partial r}+\frac{v}{r}=0, \tag{3}$$

$$v\frac{\partial u}{\partial r}+u\frac{\partial u}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial z}=0, \tag{4}$$

$$v\frac{\partial v}{\partial r}+u\frac{\partial v}{\partial z}-\frac{w^2}{r}+\frac{1}{\rho}\frac{\partial p}{\partial r}=0. \tag{5}$$

Consider now the expansion line $\psi_{zr,L}^\alpha$ as shown in Fig. 2 and define the coordinates along this line as $r=(R_2-R_1)t+R_1$ and $z=z_R t$, where $0\leqslant t\leqslant 1$. The boundary conditions for the expansion line are $u(0)=u_D$, $u(1)=0$, $v(0)=v(1)=0$. Substituting r and z in Eqs. (3)–(5) results in

$$\frac{1}{z_R}\frac{\partial u}{\partial t}+\frac{1}{R_2-R_1}\frac{\partial v}{\partial t}+\frac{v}{(R_2-R_1)t+R_1}=0, \tag{6}$$

$$\frac{1}{R_2-R_1}v\frac{\partial u}{\partial t}+\frac{1}{z_R}u\frac{\partial u}{\partial t}+\frac{1}{z_R}\frac{1}{\rho}\frac{\partial p}{\partial t}=0, \tag{7}$$

$$\frac{1}{R_2-R_1}v\frac{\partial v}{\partial t}+\frac{1}{z_R}u\frac{\partial v}{\partial t}-\frac{w^2}{(R_2-R_1)t+R_1}+\frac{1}{R_2-R_1}\frac{1}{\rho}\frac{\partial p}{\partial t}=0. \tag{8}$$

Integrating the continuity equation (Eq. (6)) from $t=0$ to 1 and taken into account the boundary conditions and

$$\int_0^1 \partial v/\partial t\,\mathrm{d}t=v(1)-v(0)=0, \tag{9}$$

$$\int_0^1 \frac{v}{(R_2-R_1)t+R_1}\mathrm{d}t \approx A\bar{v}\int_0^1\frac{1}{(R_2-R_1)t+R_1}\mathrm{d}t = A\bar{v}\frac{\ln(R_2/R_1)}{(R_2-R_1)}, \tag{10}$$

with A a proportionality factor, results in

$$-u_D+Az_R\frac{\ln(R_2/R_1)}{R_2-R_1}\bar{v}\approx 0. \tag{11}$$

The continuity equation therefore gives a scale for the mean radial velocity $\bar{v}$ as

$$\bar{v}\sim\frac{1}{A}\frac{R_2-R_1}{\ln(R_2/R_1)}\frac{u_D}{z_R}. \tag{12}$$

Integrating the momentum equation in the z-direction (Eq. (7)) results in

$$-B\frac{z_R}{R_2-R_1}\bar{v}u_D-\frac{1}{2}u_D^2+\frac{\Delta p}{\rho}\approx 0, \tag{13}$$

where B is a proportionality factor. By substituting the scale of $\bar{v}$, given by Eq. (12), into the above expression, one finds a scale for the pressure difference across the expansion line as

$$\Delta p\sim\left(\frac{C}{\ln(R_2/R_1)}+\frac{1}{2}\right)\rho u_D^2, \tag{14}$$

where $C=B/A$. Integrating the momentum equation in the r-direction (Eq. (8)) results in

$$B\frac{\bar{v}u_D}{z_R}+\frac{1}{R_2-R_1}\frac{\Delta p}{\rho}\approx\int_0^1\frac{w^2}{(R_2-R_1)t+R_1}\mathrm{d}t. \tag{15}$$

Substituting the scale for $\bar{v}$ (Eq. (12)) and Δp (Eq. (14)) into the above expression and approximating the right-hand side as

$$\int_0^1\frac{w^2}{(R_2-R_1)t+R_1}\mathrm{d}t\approx D\bar{w}^2\frac{\ln(R_2/R_1)}{R_2-R_1}, \tag{16}$$

results in

$$\frac{C}{\ln(R_2/R_1)}\left(\frac{R_2-R_1}{z_R}\right)^2u_D^2+\left(\frac{C}{\ln(R_2/R_1)}+\frac{1}{2}\right)u_D^2 \approx D\bar{w}^2\ln(R_2/R_1). \tag{17}$$

Rewriting this equation results in

$$\frac{z_R}{R_2-R_1}\sim\left(\frac{D}{C}\ln^2(R_2/R_1)\frac{\bar{w}^2}{u_D^2}-\left[1+\frac{1}{2C}\ln(R_2/R_1)\right]\right)^{-0.5}. \tag{18}$$

This means that the reattachment length decreases with increasing swirl. Moreover, Eq. (18) suggests that there may exist a singularity in the non-swirling sudden expansion which disappears with increasing swirl beyond a critical swirl number. This could explain the scatter in reattachment lengths found in the literature for non-swirling sudden expansions (Back and Roschke, 1972; Zemanick and Dougall, 1970; Morrison et al., 1988; Amano, 1982).

3. Model for the reattachment length

Let us now define a cartesian coordinate system such that the origin is situated on the central axis, which is coin-

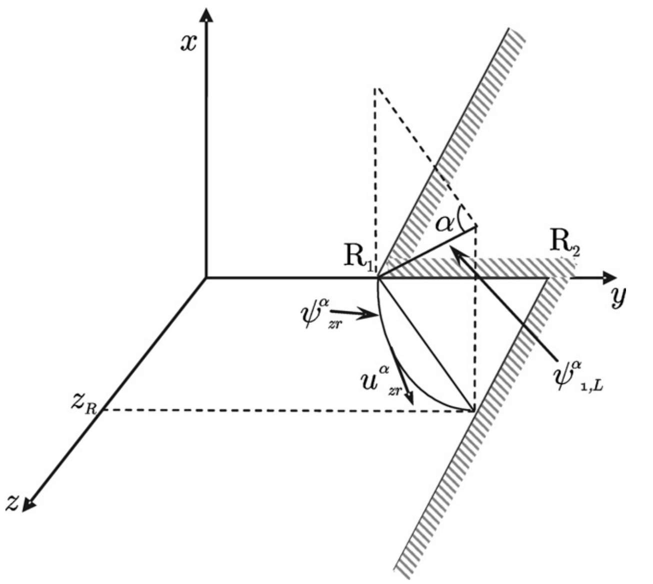

Fig. 4. Expansion line for swirling flow. Only the cross-sections of the upstream and downstream channel with the (y,z) plane are shown.

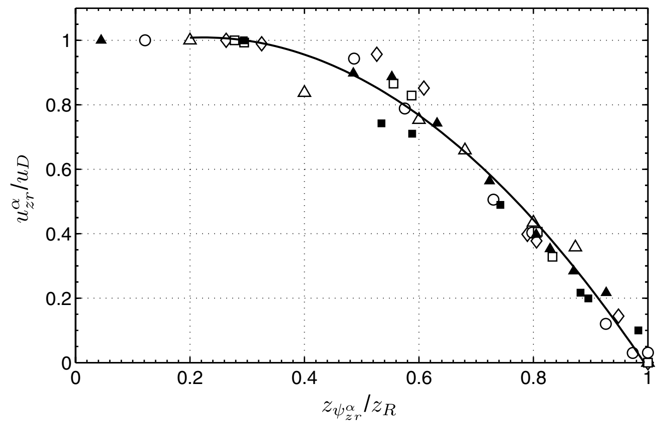

Fig. 5. The normalised axial velocity along the projected streamline ψ^{α}_{zr}. Data from Dellenback (1986). ○: $S=0$, $Re=100{,}000$; △: $S=0.6$, $Re=30{,}000$; ◇: $S=0.98$, $Re=30{,}000$; □: $S=1.23$, $Re=30{,}000$. Data from Wang et al. (2004). ▲: $S=0.33$, $Re=10{,}000$; ■: $S=0.43$, $Re=20{,}000$. Solid line: 2nd order regression line with $R^2=0.97$.

cident with the z-axis of the coordinate system (Fig. 4). A randomly chosen detachment point on the edge of the upstream channel lies on the y-axis with coordinates $(0, R_1, 0)$. Define now an expansion line $\psi_{L,1}$ which connects the detachment point with the reattachment point in a non-swirling flow. The parametric equation for such a line is

$$\psi_{1,L}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ R_1 \\ 0 \end{pmatrix} + k\begin{pmatrix} 0 \\ \frac{1}{\sqrt{1+\delta_0^2}} \\ \frac{\delta_0}{\sqrt{1+\delta_0^2}} \end{pmatrix}, \tag{19}$$

where $k \in [-\infty, \infty]$ and $\delta_0 = z_{R,0}/(R_2 - R_1)$ is the ratio of the reattachment length to the height of the sudden expansion. In fact $\Psi_{1,L}$ is a ruled surface with $\psi_{1,L}$ as the ruler and the inner edge of the sudden expansion as the base curve. It is found in the literature that the detachment point is Reynolds number independent and therefore also $\psi_{1,L}$.

The addition of swirl to the upstream channel flow creates a rotational velocity component superimposed on the axial and radial velocity components. This swirl component rotates the streamline out of the (y,z)-plane by an angle $\alpha = \arctan S_{\Psi^{\alpha}_1}$ (Fig. 4). It follows from Fig. 3 that this angle remains constant till $z_{\psi^{\alpha}_1}/z_R \leqslant 0.6$. This means that for $z_{\psi^{\alpha}_1}/z_R \leqslant 0.6 \psi^{\alpha}_1$ is a 2D curve and its projection $\psi^{\alpha}_{\mathbf{C}_2}$ is a line. Consider now the projection of the velocity $u_{\psi^{\alpha}_1}$ on ψ^{α}_{zr} defined as $u^{\alpha}_{zr} = \mathscr{P}_{\psi^{\alpha}_{zr}}(u_{\psi^{\alpha}_1})$. Fig. 5 shows the normalised velocity u^{α}_{zr}/u_D plotted against the dimensionless z-coordinate $z_{\psi^{\alpha}_{zr}}/z_R$. This figure shows that this projection is invariant to the swirl number. Because of this similarity between the flow fields at zero and non-zero swirl, as depicted by Fig. 5, the projection $\psi^{\alpha}_{\mathbf{C}_2}$ can be approximated by the projection of $\psi^{\alpha}_{1,L}$ on $\mathbf{C}_2$, where $\psi^{\alpha}_{1,L}$ is a rotation of $\psi_{1,L}$ due to the swirl. The equation of $\psi^{\alpha}_{1,L}$ therefore becomes

$$\psi^{\alpha}_{1,L}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ R_1 \\ 0 \end{pmatrix} + k\begin{pmatrix} \sin\alpha_D \\ \frac{\cos\alpha_D}{\sqrt{1+\delta_0^2}} \\ \frac{\delta_0\cos\alpha_D}{\sqrt{1+\delta_0^2}} \end{pmatrix}. \tag{20}$$

In this approach, $\psi_{1,L}$ serves as a calibration line for extrapolation to flows with swirl.

The z-coordinate of the reattachment point can be determined by solving the intersection between $\psi^{\alpha}_{1,L}$ and $\mathbf{C}_2$, which is a surface representing the downstream channel. The equation for the surface $\mathbf{C}_2$ is given by

$$\mathbf{C}_2(x,y,z) : x^2 + y^2 = R_2^2, \quad z \geqslant 0. \tag{21}$$

By substituting Eq. (20) in Eq. (21), an expression for k can be found:

$$k = R_1\sqrt{1+\delta_0^2} \times \left(\frac{-\cos\alpha_D \pm \sqrt{h^2(1+\delta_0^2\sin^2\alpha_D) - (1+\delta_0^2)\sin^2\alpha_D}}{1+\delta_0^2\sin^2\alpha_D}\right), \tag{22}$$

with $h = R_2/R_1$ the expansion ratio. Since the axial coordinate z has to be positive in Eq. (21) the + sign in Eq. (22) is taken. The reattachment length is equal to $z_R = k\delta_0\cos\alpha_D/\sqrt{(1+\delta_0^2)}$ and therefore

$$z_R = R_1\delta_0\cos\alpha_D \times \left(\frac{-\cos\alpha_D \pm \sqrt{h^2(1+\delta_0^2\sin^2\alpha_D) - (1+\delta_0^2)\sin^2\alpha_D}}{1+\delta_0^2\sin^2\alpha_D}\right). \tag{23}$$

Substituting $\tan\alpha_D = S_D$, $\cos\alpha_D = (\sqrt{1+\tan^2\alpha_D})^{-1}$ and $\sin\alpha_D = \tan\alpha_D/\sqrt{1+\tan^2\alpha_D}$ into Eq. (23) gives an analytical expression for z_R as a function of S_D,

$$z_R = \frac{R_1\delta_0}{1+(1+\delta_0^2)S_D^2}\left(\sqrt{h^2(1+(1+\delta_0^2)S_D^2)-(1+\delta_0^2)S_D^2}-1\right). \tag{24}$$

Eq. (24) can be simplified since $1+\delta_0^2 \approx \delta_0^2$, which yields the final expression for the reattachment length with swirl,

$$z_R = \frac{R_1\delta_0}{1+\delta_0^2 S_D^2}\left(\sqrt{(h^2-1)\delta_0^2 S_D^2 + h^2}-1\right). \tag{25}$$

Comparison with the scale analysis shows that for high swirl numbers the reattachment length is inversely proportional to the swirl number or $z_R \sim 1/S_D$.

4. Validation of the model

Validation of the model is based on the measurements of Dellenback et al. (1988), Wang et al. (2004), Favaloro et al. (1989) Ahmed and Nejad (1992) and Nejad and Ahmed (1992). In the literature the most common parameter to describe swirling flow is the momentum flux averaged swirl number S, defined as

$$S = \frac{\int_0^{R_1} 2\pi\rho U W r^2\,dr}{R_1\int_0^{R_1} 2\pi\rho U^2 r\,dr}. \tag{26}$$

This dimensionless swirl number is the ratio of axial flux of tangential momentum divided by the axial flux of axial momentum and outer radius. The relation between the local swirl number S_D and S is determined using quantitative data provided by Dellenback (1986) and Wang et al. (2004). Based on their measured velocity fields the stream-surfaces Ψ^α are calculated. The values of $w^\alpha_{\psi_1}$ and $u^\alpha_{\psi_1}$ along the detaching streamline ψ_1^α are calculated using spline interpolation. The result is shown in Fig. 6. A regression line of 2nd order is fitted through the experimental data. Favaloro et al. (1989) reported only reattachment lengths as a function of the swirl number. Since they use the same type of swirl generator as Wang et al. did, the local swirl ratio is determined using the curve in Fig. 6. The measurement errors of the reported experimental data of Favaloro are in the same order as the ones of Wang et al. Therefore, the relative error of S_D is estimated to be the same as Wang, namely 6%. Both Ahmed and Nejad (1992) and Nejad and Ahmed (1992) reported velocity profiles at $z/(R_2-R_1) = 0.38$. The local swirl number is estimated based on the reported data and the absolute error is 0.05.

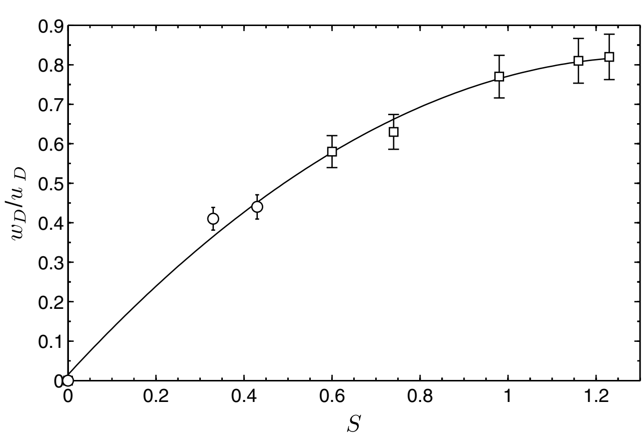

Fig. 6. The local swirl number S_D as a function of the swirl number S. □, data from Dellenback (1986); ○, data from Wang et al. (2004).

Table 1 shows the experimental and computed values of $z_R/(R_2-R_1)$ of the different researchers. The computed local swirl number S_D is shown between brackets together with the lower and upper boundaries of the 95% confidence interval. The lower and upper limit of the predictions of the model are based on those boundaries of S_D. Only Dellenback reported measurement errors. The experimentally measured reattachment length was determined using an average of 4 to 8 measurements and the accuracy reported was ±0.5 step height in the non-swirling case and ±0.1 step height in the swirling cases. The measured reattachment lengths are smaller than predicted by the model and are situated near the lower part of the model boundaries. However, taking into account the accuracy of the measured values the confidence intervals of the measured and predicted values overlap in all the swirl cases. The measured data by Wang et al. (2004) are higher than the predicted values of the model. There were no accuracies reported so it is impossible to say wether the confidence intervals overlap. The measured values of Favaloro et al. (1989) are situated very close to the lower boundaries of the predicted values. There were also no measurement errors reported. However, it is very likely that the confidence intervals of predicted and measured values overlap.

The reattachment length of Dellenback seems to go to a constant value for very high swirl numbers. Looking at S_D in Fig. 6 shows that this local swirl number remains more or less constant for those high swirl numbers. This indicates that the swirl number S is not an all inclusive parameter for prediction of the reattachment length. Instead it is better to use the local swirl ratio S_D to characterise the reattachment length. Studies that confirm this statement are the numerical study of Wang and Bai (2005) and the experimental studies of Ahmed and Nejad (1992) and Nejad and Ahmed (1992). The first one by Wang and Bai (2005) investigated numerically the swirling flow in a dump combustor using LES. One aspect of investigation was the influence of the swirl velocity profile. The axial velocity profile was kept constant and 3 different swirl profiles were applied in such way that the swirl number was the same for all three cases. It was shown that the swirl profile with the largest swirl velocity near the outer wall of the upstream channel showed a significantly decrease in reattachment length (from 3 to 2.25). Because of the higher swirl velocity near the outer wall, this profile has a larger local swirl number S_D. Unfortunately no profiles were given which made it impossible to calculate the local swirl number. The second

Table 1
Comparison of the model with data found in the literature

Data	h [–]	Re [–]	S [–]	S_D	$\frac{z_R}{R_2-R_1}$ [–]	$\frac{z_R}{R_2-R_1}$ model [–]
A	1.94	30,000	0	0	9.3	
			0.6	0.54 (0.58) 0.62	2.5	2.52–2.83
			0.98	0.72 (0.77) 0.82	1.9	1.96–2.21
		60,000	0	0	9.2	
			0.77	0.63 (0.68*) 0.73	2.2	2.19–2.47
			1.16	0.75 (0.81) 0.87	1.8	1.87–2.11
		100,000	0	0	9.0	
			0.74	0.59 (0.63) 0.67	2.2	2.34–2.64
			1.23	0.76 (0.82) 0.88	1.8	1.85–2.09
B	2	10,000	0	–	10.3	–
		10,000	0.33	0.39 (0.41) 0.43	3.79	3.52–3.81
		20,000	0.43	0.41 (0.44) 0.47,	3.57	3.26–3.66
C	1.5	150,000	0	–	8.1	–
			0.3	0.33 (0.34*) 0.35	4.3	4.34–4.54
			0.5	0.49 (0.51*) 0.53	3.2	3.24–3.41
D	1.5	125,000	0	–	8.25	–
			0.3	0.35 (0.4†) 0.45	4.3	3.69–4.41
			0.4	0.45 (0.5†) 0.55	3.55	3.16–3.69
			0.5	0.55 (0.6†) 0.65	3.38	2.75–3.16
E	1.5	125,000	0	–	8.25	–
			0.199	0.45 (0.5†) 0.55	3.55	3.16–3.69
			0.205	0.65 (0.7†) 0.75	2.8	2.44–2.75
			0.249	0.35 (0.4†) 0.45	4	3.69–4.41

Data A: Dellenback et al. (1988), data B: Wang et al. (2004), data C: Favaloro et al. (1989), data D: Ahmed and Nejad (1992) and data E: Ahmed and Nejad (1992) and Nejad and Ahmed (1992). * Ratio calculated using curve in Fig. 6, † ratio calculated using the reported velocity profiles at $z/(R_2 - R_1) = 0.38$.

study is performed by Ahmed and Nejad (1992) and Nejad and Ahmed (1992). In contrast to the numerical study of Wang and Bai (2005), their studies are experimental. They used three different swirl generators, each with a different swirl inlet profile. The swirl generators that are used are a 'Forced vortex' (FOV), a 'Free Vortex' (FRV) and a 'Constant Angle' (CA) generator. Table 2 summarises the results. The local swirl number is estimated based on the reported velocity profiles at $z/(R_2 - R_1) = 0.38$. This table shows that the local swirl number is a better parameter to characterise the flow, since there is a monotonic decrease in reattachment length with increasing local swirl number. Despite the fact that the swirl number of the FRV is 20% larger than the FOV, the reattachment length of the FRV is 36% longer, which is unexpected since the reattachment length should increase with decreasing swirl. These three studies show that the swirl number is not an all inclusive parameter to characterise the reattachment length. The inlet swirl profile is also an important parameter and the local swirl number takes into account this swirl distribution to a better extend than the swirl number.

Table 2
Summary of the experimental results of Ahmed and Nejad (1992) and Nejad and Ahmed (1992)

Swirl generator	FRV	CA	FOV
S	0.249	0.199	0.205
S_D	0.4	0.5	0.7
$z_R/(R_2 - R_1)$	4	3.55	2.8

FOV, FOrced Vortex swirl generator; FRV, FRee Vortex swirl generator; CA, Constant Angle swirl generator.

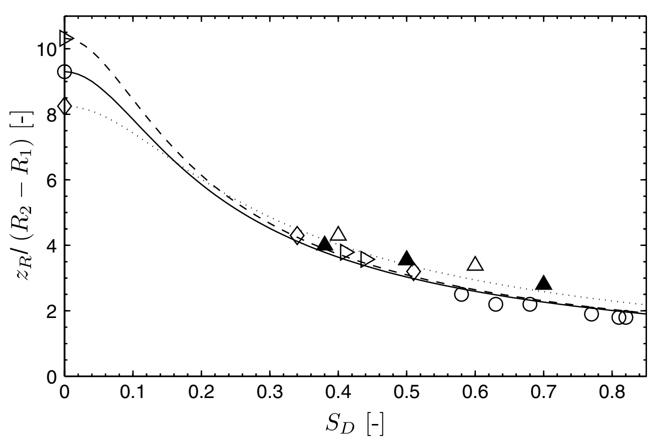

Fig. 7. Predictions of the reattachment length in number of step heights $z_R/(R_2 - R_1)$ a function of the local swirl number S_D compared with the literature. Solid line: model with δ_0 from Dellenback et al. (1988), dashed line: model with δ_0 from Wang et al. (2004) and dotted line: model with δ_0 from Favaloro et al. (1989), Nejad and Ahmed (1992), Ahmed and Nejad (1992); ▷: data from Wang et al. (2004); ◇: data from Favaloro et al. (1989) ○: data from Dellenback et al. (1988), ▲: data from Ahmed and Nejad (1992), Nejad and Ahmed (1992) in Table 2; △: data from Ahmed and Nejad (1992).

Fig. 7 shows the curves of Eq. (25). The solid line represents the theoretical model based on the expansion ratio and δ_0 of Dellenback (1986) the dashed line represents the model based on the expansion ratio and δ_0 of Wang et al. (2004) and the dotted line represents the model based on the expansion ratio and δ_0 of Favaloro et al. (1989). All three researchers measure a strong decrease of the reattachment length with increasing swirl. At zero swirl the different researchers found reattachment lengths which can vary more than 2 step heights. This indicates the strong turbulent nature of the flow and the sensitivity of the reattachment length to parameters such as the inlet velocity profile and turbulence intensity and the existence of pressure gradients in the upstream flow (Kuehn, 1980; Eaton and Johnston, 1980). These differences seems to decrease with increasing swirl which explains the smaller deviation between the researchers for higher swirl numbers. More influential than the reattachment length at zero swirl is the expansion ratio $h = R_2/R_1$. The model predicts a lower decrease of reattachment length with a smaller expansion ratio. However, no studies in the literature exist to the authors knowledge which could experimentally or numerically verify this prediction.

5. Concluding remarks

An analytical expression has been derived to predict the influence of swirl on the reattachment length in an axisymmetric sudden expansion. This expression is calibrated with non-swirl reattachment data and uses the similarity between swirling and non-swirling flows to account for the swirl. It is found that the reattachment length depends on the swirl number S, the expansion ratio R_2/R_1 and, to some extend, on the reattachment length at zero swirl. A better parameter to describe the reattachment length is the local swirl number S_D of the detaching streamline instead of the swirl number S. The reason is the important role of the swirl velocity profile on the reattachment length. The local swirl number takes into account this distribution better than the momentum flux averaged swirl number and is hence a better parameter to characterise the reattachment length. The relation between S and S_D is based on selected experimental results. The theoretical model is validated by comparison with experimental measurements found in the literature for Reynolds numbers ranging from 10,000 to 100,000 and expansion ratios from 1.5 to 2. From the model it also follows that swirl has a stabilising effect on the reattachment length which results in smaller deviations in the reattachment length between experimental data for swirling flows compared to non-swirling flows. This is confirmed by a scaling analysis which suggests there might be a singularity for non-swirling flows in the reattachment length, which disappears when the swirl is increased beyond a certain critical value. This possible singularity could explain the deviation in reattachment lengths found in the literature for non-swirling sudden expansion flows.

Acknowledgements

The authors gratefully thank Prof. P.A. Dellenback and Prof. X.S. Bai for providing us their quantitative experimental data for validation of the model.

References

Abujelala, M.T., Lilley, D.G., 1984. Swirl, confinement and nozzle effects on confined turbulent flow. AIAA/SAE/ASME 20th Joint Propulsion Conference, June 11–13, Cincinnati, OH.

Ahmed, S.A., Nejad, A.S., 1992. Swirl effects on confined flows in axisymmetric geometries. Journal of Propulsion and Power 8 (2), 339–345.

Ahmed, S.A., 1998. Velocity measurements and turbulence statistics of a confined isothermal swirling flow. Experimental Thermal and Fluid Science 17, 256–264.

Amano, R.S., 1982. A study of turbulent flow downstream of an abrupt pipe expansion. AIAA Journal 21 (10), 1400–1405.

Armaly, B.F., Durst, F., Pereira, J.C.F., Schoning, B., 1983. Experimental and theoretical investigation of backward-facing step flow. Journal of Fluid Mechanics 127, 473–496.

Back, L.H., Roschke, E.J., 1972. Shear-layer flow regimes and wave instabilities and reattachment lengths downstream of an abrupt circular channel expansion. Journal of Applied Mechanics 39, 677–681.

Dellenback, P.A., 1986. Heat Transfer and Velocity Measurements in Turbulent Swirling Flow Downstream of an Abrupt Axisymmetric Expansion, Ph.D. Dissertation, Arizona State University, Tempe, AZ.

Dellenback, P.A., Metzger, D.E., Neitzel, G.P., 1988. Measurements in turbulent swirling flow through an abrupt axisymmetric expansion. AIAA Journal 26 (6), 669–681.

Eaton, J.K., Johnston, J.P., 1980. Turbulent Flow Reattachment. Mechanical Engineering Department, Stanford University, Stanford, CA, Rept. MD-39.

Favaloro, S.C., Nejad, A.S., Ahmed, S.A., 1989. Experimental and computational investigation of isothermal swirling flow in an axisymmetric dump combustor. Journal of Propulsion 7 (3), 348–356.

Garkusha, A.V., Kucherenko, S.I., 1980. Investigation of swirling flow in a circular channel with sudden expansion. Fluid Dynamics (Historical Archive) 15 (5), 766–771.

Gupta, A., Lilley, G., Syred, N., 1984. Swirl Flows. Abacus Press, Tunbridge Wells, Kent, UK.

Hallett, W.L.H., Gunther, R., 1984. Flow and mixing in swirling flow in a sudden expansion. The Canadian Journal of Chemical Engineering 62, 149–155.

Kuehn, D.M., 1980. Effects of adverse pressure gradient on the incompressible reattaching flow over a rearward-facing step. AIAA Journal 18, 343–344.

Morrison, G.L., Tatterson, G.B., Long, M.W., 1988. Three-dimensional laser velocimeter investigation of turbulent, incompressible flow in an axisymmetric sudden expansion. Journal of Propulsion 4 (6), 533–540.

Nejad, A.S., Ahmed, S.A., 1992. Flow field characteristics of an axisymmetric sudden-expansion pipe flow with different initial swirl distribution. International Journal of Heat and Fluid Flow 13 (4), 314–321.

Rhode, D.L., Lilley, D.G., Mc Laughlin, D., 1983. Mean flowfields in axisymmetric combustor geometries with swirl. AIAA Journal 21 (4), 593–600.

Syred, N., 2006. A review of oscillation mechanisms and the role of the precessing vortex core (PVC) in swirl combustion systems. Progress in Energy and Combustion Science 32, 93–161.

Wang, P., Bai, X.S., Wessman, M., Klingmann, J., 2004. Large eddy simulation and experimental studies of a confined turbulent swirling flow. Physics of Fluids 16 (9), 3306–3324.

Wang, P., Bai, X.S., 2005. Large eddy simulations of turbulent swirling flows in a dump combustor: a sensitivity study. International Journal for Numerical Methods in Fluids 47, 99–120.

Zemanick, P.P., Dougall, R.S., 1970. Local heat transfer downstream of an abrupt circular channel expansion. Transactions of the ASME C: Journal of Heat Transfer 92 (1), 53–60.